CHAPTER FIVE

FUNCTIONS AND ITS ASSOCIATED SIMPLIFICATION

Simplification:

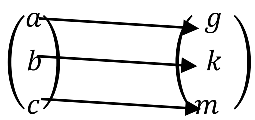
- Let x and y be two sets. When each number of the set x is associated or related to only one member of the set *y*, then such a relation is known as a function from x to y.
- This is written as f: x → *y* and read as "the function from the set x to the set y or by the equation y = f(*x*).
- The set x is known as the domain and the set y is known as the co-domain or the images.
- The word function emphasizes the idea of the dependence of one quality on another. For example, let f be the mapping which is defined by f: $x \rightarrow 2x+1$, which can be written as $y = 2x + 1$. We say that y is a function of x which means that y depends on x
- The variable x is called the independent variable, and y is called the dependent variable. The type of relation between x and y is called a functional relation. Each of the following defines the same set.

1) F: $\{x \rightarrow 2x - 1, x \in N\}$.
2) F = $\{(x,y): y = 2x - 1, x \in N\}$.
3) F = $\{x, 2x - 1: x \in N\}$.
4) $Y = 2x - 1, x \in N$.
5) $F(x) = 2x - 1, x \in N$.

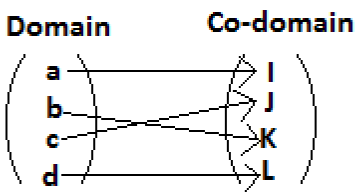
A function (or mapping) is therefore the relation between the elements of two sets, which are the domain and the co-domain, such that each element within the domain is associated or related to only one element in the co-domain.

Example (1)

Domain Co-domain



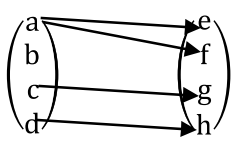
Example (2)



This is also a function, since each member of the mdomain is associated with only one member of the co-domain.

Example (3)

Domain Co-domain



This is not a function, for the first member of the domain i.e a, is associated with two members of the co-domain.

(Q1.) Given that F(x) = 2x+1, evaluated the following:

f(2) (b.) f(4) (c.) f(-3)

(d) f(-1) (e.) 2f(x) (f.) 5f(x).

<u>Soln.</u>

F(x) = 2x+1 =>

a.F(2) = 2(2)+1 = 4+1 = 5.

b. F(4) = 2(4)+1 = 8+1 = 9.

c. F(-3) = 2(-3)+1 = -6+1 = -5.

d. F(-1) = 2(-1)+1 = -2+1 = -1.

e. Since f(x) = (2x+1) $\Rightarrow$ 2f(x) = 2(2x+1) = 4x+2.

f. 5f(x) = 5(2x+1) = 10x +5.

N/B: F(x) = 2x + 1 can be written as F(x) = (2x + 1) or F(x) = 1(2x+1).

(Q2.) If g(x) = 3x – 1, evaluate the following:

a.g(-1)   b.) g(-2)   c.) g($^1/_2$)

d.) 3g(x) +1   e.) 4 g(x) – 2

f.) -2g(x)+2   g.) -3g(x) -3.

<u>Soln.</u>

g(x) = 3x – 1 =>

a.g(-1) = 3(-1) -1 = -3 – 1 = -4.

b. g(-2) = 3(-2) -1 = - 6 – 1 = -7.

c. g($^1/_2$) = 3($^1/_2$) -1 = 3 x $^1/_2$ – 1 = 1.5 – 1 = 0.5.

d. g(x) = 3x – 1 => 3g(x) + 1 = 3(3x-1) + 1 = 9x – 3 +1 = 9x – 2 .

e. g(x) = 3x – 1 => 4 g(x) -2 = 4(3x - 1) -2 = (12x - 4) -2 = 12x - 4 - 2 = 12x – 6.

f. g(x) = 3x – 1 => -2 g(x) + 2 = -2(3x-1) + 2 = (- 6x+2) +2 = - 6x+2+2 = - 6x+4.

g. g(x) = 3x – 1 => -3g(x) -3 = -3(3x – 1) -3 = (-9x +3) -3 = -9x+3 -3 = -9x.

Q3. Given that f(x) = 2x + 1 and g(x) = 4x +2, evaluate the following:

a. g(x) +f(x)   b. 2g(x) +f(x)   c. 3g(x) + 4f(x)

d. ½ g(x) + 2f(x)   e. g(x) – f(x)   f. 3g(x) – 2(fx) soln.

$g(x) = 4x+2$ and $F(x) = 2x+1$ =>

a.) $g(x) + f(x) = (4x+2) + (2x+1) = 6x+3$.

b.) $2g(x) + f(x) = 2(4x+2) + (2x+1) = 8x+4+2x+1 = 8x+2x+4+1 = 10x +5$

c.) $3g(x) + 4f(x) = 3(4x+2) + 4(2x+1) = (12x+6) + (8x+4) = 12x +6+8x + 4 = 12x+8x+6+4 = 20x+10$.

d.) $\frac{1}{2} g(x) +2f(x) = \frac{1}{2}(4x+2) +2 (2x+1) = \frac{1}{2} \times 4x+1/2 \times 2 +4x +2 = 2x+1+4x+2 = 2x + 4x + 1+2 = 6x +3$.

e.) $g(x) – f(x)$

$= (4x +2) – (2x + 1) = 4x+2 – 2x – 1$,

$= 4x – 2x + 2 – 1 = 2x +1$.

f.) $3g(x) – 2f(x)$

$= 3(4x + 2) -2(2x +1)$,

$= 12x + 6 - 4x – 2 = 12x – 4x + 6 – 2$

$= 8x + 4$.

Q4. Given that $f(x) = 3x + 2$ and $g(x) = -4x – 2$, evaluate the following.

a.) (i) $f(-1)$ (ii) $f(-2)$

b.) (i) $g(-1)$ (ii) $g(-2)$ (iii) $g(2)$

c.) (i) $f(x) + g(x)$ (ii) $f(x) – g(x)$

d.) (i) $2f(x) + 3$ e.) $3f(x) – 2$

f.) $g(x) – f(x)$

Soln.

a.) $f(x) = 3x + 2 \Rightarrow$

(i) $f(1) = 3(1) + 2 = 3 + 2 = 5$.

(ii) $f(-2) = 3(-2) + 2 = -6 + 2 = -4$.

b.) $g(x) = -4x – 2 \Rightarrow$

(i) $g(-1) = -4(-1) – 2 = 4 – 2 = 2$.

(ii) $g(-2) = -4(-2) - 2= 8 – 2 = 6$.

(iii) $g(2) = - 4(2) – 2 = -8 – 2 = -10$.

c.) (i) $f(x) + g(x) = (3x + 2) + (- 4x – 2)$

$= 3x + 2 – 4x – 2 = 3x – 4x + 2 – 2$

$= -x + 0 = -x$

(ii) $f(x) – g(x) = (3x + 2) – (- 4x – 2)$

$= 3x + 2 + 4x + 2 = 3x + 4x + 2 + 2$

$= 7x + 4$.

d.) $2f(x) + 3 = 2(3x + 2) + 3$

$= 6x + 4 + 3 = 6x + 7$.

e.) $3f(x) – 2 = 3(3x + 2) – 2 = 9x + 6 – 2$

$= 9x + 4$.

f.) $g(x) – f(x) = (-4x – 2) – (3x + 2)$

$= - 4x – 2 – 3x – 2 = - 4x – 3x – 2 – 2$

$= -7x – 4$.

Q6. Given that $g(x) = x – 2$, evaluate the following:

a. $g(3x+1)$ b. $g(-2x+1)$

c. $g(-4x – 3)$ d. $g(2x – 1)$

Soln.

a. $g(x) = x – 2 \Rightarrow g(3x+1) = (3x+1) - 2 = 3x+1 – 2$

$= 3x – 1.$

b. $g(x) = x- 2 \Rightarrow g(-2x+1)$

$= (-2x+1) – 2 = -2x + 1 – 2 = -2x – 1.$

c. $g(x) = x – 2 \Rightarrow g(-4x – 3) = (-4x – 3) -2 = (-4x – 3) – 2$

$= -4x – 3 – 2 = -4x – 5.$

d. $g(x) = x – 2 \Rightarrow g(2x – 1) = (2x – 1) – 2 = 2x – 3.$

Q7. Given that $f(x) = 2x – 1$ and $g(x) = x^2 + 1$, find

a. (i) $f(1 + x)$ (ii) $f(3x – 1)$

b. Simplify $f(x) – g(x)$.

c. Simplify $3f(x) + 2g(x)$.

d. Determine the values of x for which

(i) $f(x) > 5$ (ii) $f(x) \geq 7$ (iii) $f(x) < -3$ (iv) $f(x) \leq -9$

Soln.

a. (i) $f(x) = 2x – 1 \Rightarrow f(1 + x) = 2(1 + x) – 1 = 2 + 2x – 1 = 2x + 2 – 1 = 2x + 1.$

(ii) $f(x) = 2x – 1 \Rightarrow f(3x – 1) = 2(3x – 1) –1 = 6x – 2 – 1 = 6x – 3.$

b. $f(x) – g(x) = (2x – 1) – (x^2 + 1)$

$= 2x – 1 - x^2 – 1 = 2x – x^2 – 1 – 1$

$= 2x – x^2 – 2 = -x^2 + 2x - 2.$

c. $3f(x) + 2g(x) = 3(2x – 1) + 2(x^2 + 1)$

$= 6x – 3 + 2x^2 + 2 = 6x + 2x^2 – 3 + 2$

$= 6x + 2x^2 – 1 = 2x^2 + 6x - 1.$

d. $f(x) = 2x - 1$. Since $f(x) > 5 \Rightarrow 2x - 1 > 5$,

$\Rightarrow 2x > 5 + 1 \Rightarrow 2x > 6 \Rightarrow x > \frac{6}{2}$

$\Rightarrow x > 3$.

(ii) If $f(x) \geq 7 \Rightarrow 2x - 1 \geq 7$

$\Rightarrow 2x \geq 7 + 1, \Rightarrow 2x \geq 8 \Rightarrow x \geq {}^{8}/_{2}$,

$\Rightarrow x \geq 4$.

(iii) $f(x) < -3, \Rightarrow 2x - 1 < -3$

$\Rightarrow 2x < -3 + 1, \Rightarrow 2x < -2$

$\Rightarrow x < \frac{-2}{2}, \Rightarrow x < -1$.

(iv) $f(x) = 2x - 1$. Since $f(x) \leq -9$

$\Rightarrow 2x - 1 \leq 9 \Rightarrow 2x \leq 9 + 1$,

$\Rightarrow 2x \leq 10 \Rightarrow x \leq \frac{10}{2} \Rightarrow x \leq 5$.